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FACTORIZATION AND UNIVERSALITY
IN AZIMUTHAL ASYMMETRIES[*]

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The theoretical status on transverse momentum dependent factorization in semi-inclusive DIS and Drell-Yan is not clear in contrast to claims in the literature in which gauge links only at one loop were considered explicitly. Recently obtained results beyond this order question the validity of these claims and will be briefly discussed. Possible input from experiments to solve these matters will be outlined.

1. The present theoretical status

Single spin asymmetries (SSA's) are full of surprises and give rise to questions about factorization. In a factorized description of SSA's one needs a special class, called T-odd, of *transverse momentum dependent* distribution functions or fragmentation functions. Three separate mechanisms were suggested to generate T-odd functions. The first mechanism[1] consists of nonzero gluon fields at infinity and was unified[2] with the second mechanism which is based on fully connected gauge links[3]. The non-trivial paths of these links connect the two quark fields in the distribution and fragmentation functions and could lead to SSA's. The third mechanism, appearing only for fragmentation functions, comes from final state interactions[4].

In a factorized description the gauge links have particular implications: T-odd distribution functions in semi-inclusive DIS (SIDIS) enter with a different sign in Drell-Yan (DY)[5]; T-odd distribution functions involve the gluon field in the nucleon[2]; gauge links violate naive Lorentz-invariance relations[6]; links can give rise to new functions[7]; and, links might imply non-universal fragmentation functions[2]. This questions whether a factorized description is allowed for transverse momentum dependent observables.

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Several articles deal with factorization in SIDIS and DY. Recently a spin dependent factorization theorem for SIDIS and DY was claimed where the all-order argument is based on generalized Ward identities[8]. Subsequently, part of the relevant all-order calculations were presented[9] and possible problems on factorization in DY were pointed out[10]. Afterwards, it was suggested that the fragmentation functions and soft factor in the factorization theorem should contain different gauge links and the authors[11] claimed universality of distribution, fragmentation functions and soft factors in SIDIS, e^+e^- annihilation and DY. The proof of universal fragmentation functions was formalized at one loop, but again the all-order arguments are based on Ward identities and an explicit proof was not given.

Despite the claims on factorization and the significant progress made in the previous references, the present situation is not clear. However, factorization remains essential for comparing experimental results and relating them to theoretical predictions. In the next section gauge link derivations[9] and their consequences for factorization[10] will be briefly presented. As will be shown, Ward identities should be applied carefully. The last section will discuss how experiments and theory could contribute to solve these matters.

2. Gauge links and factorization

Gauge links for hard scattering diagrams can be derived by coupling on-shell longitudinally polarized gluons to the diagrams. The delicate use of Ward identities in these calculations is illustrated by the following QED example. When considering S-matrix elements the sum of coupling a photon with $\epsilon = p_1$ (and momentum $p_1 \sim n_+$) to all possible places in the tree-level amplitude $\gamma^*(q - p_1) + e(k) \rightarrow e(p)$ vanishes, giving the relation

$$\text{(diagram)} \quad = \quad - \; \text{(diagram)} \quad . \tag{1}$$

However, in link derivations where the photon polarization is the momentum *direction*, the sum does not vanish. In fact, the sum equals a gluonic pole matrix element[2,9] which could produce a SSA by itself

$$\text{(diagram)} \quad + \quad \text{(diagram)} \quad = \quad \text{(diagram)} \quad \left(\frac{1}{p_1^+ + i\epsilon} + \frac{1}{p_1^+ - i\epsilon} \right). \tag{2}$$

Since similar effects also appear in QCD we refrained from using identities like Eq.(1). Summing over the gluons explicitly the link is straightforwardly derived to all orders in the coupling and allows for easy consistency checks
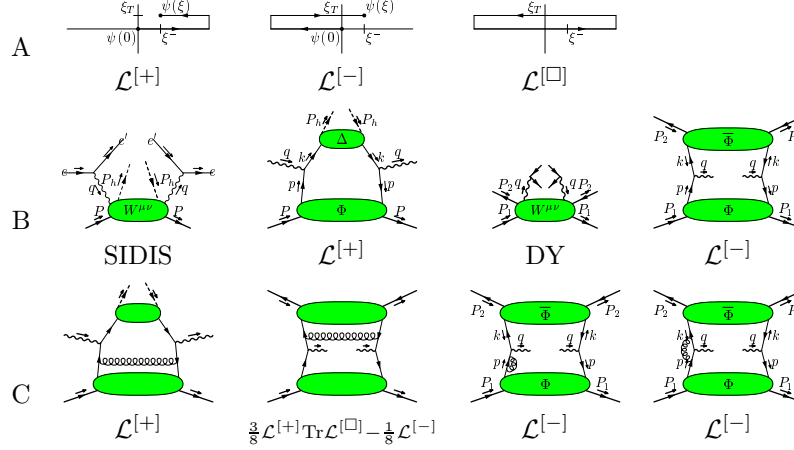
Figure 1.   The gauge links ($\mathcal{L}$) presented here are obtained as follows: (1) start with a certain hard scattering diagram with correlators containing no links, (2) sum over all diagrams of longitudinally polarized gluons connecting a considered soft blob and the hard diagram in the eikonal approximation. Gluons coupling directly to the quark of the considered correlator are already present in the soft blob definition, (3) the sum results in a link in the considered soft blob multiplied by the *same* hard scattering diagram.
A: various gauge links. B: tree-level SIDIS and DY, cross-sections and leading contributions in parton model with gauge links for Φ. C: corrections to SIDIS and DY with gauge links for lower blobs; the coupling of the longitudinally polarized gluons to the explicitly drawn gluon *has to be included* to obtain a proper gauge link; the gauge links for the virtual corrections have only be verified to the first non-trivial order.

by others (are the first orders of the link correct and is the quark-quark correlator gauge invariant?). Some obtained results[2,9,10] are given in Fig.1.

Gauge links form an essential ingredient in considerations on factorization. In factorization one typically tries to absorb gluon radiation with small transverse momentum in a soft blob hoping that the constructed soft blobs are in some sense universal. From Fig.1C it becomes clear that the behaviour of gauge links when gluons are radiated depends on the process. Although still calculable, the radiated gluon in DY in Fig.1C2 needs to be absorbed when constructing the upper blob, but since the gluon affects the gauge link of the lower blob, it will be difficult - if not impossible - to factorize such diagrams[10]. This result appears beyond the explicitly considered one-loop calculations of the earlier discussed references.

Similar effects appear in other hadron-hadron scattering processes and in fragmentation functions in SIDIS. They do not appear in distribution functions in lepton and photon-hadron scattering, and in fragmentation functions in e^+e^- annihilation.

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3. Experimental and theoretical input

We would like to stress that in those processes where one is not sensitive to the transverse momenta of the quarks, one is dealing with transverse momentum integrated distribution and fragmentation functions. The links connect the two quark fields by a straight line and are process independent. Therefore, integrated SIDIS and DY have no problems regarding their link structures. As such, transversity can be best accessed via integrated DY, Λ polarization in SIDIS or two hadron fragmentation in SIDIS[12].

To understand transverse momentum dependent factorization and universality we need experimental and theoretical input. A comparison of a T-odd distribution function, such as the Sivers function, in SIDIS and DY can have the following outcomes: (1) they only differ by a sign and the processes apparently factorize and links have predicting power, (2) they are both zero (for some unknown symmetry), a factorization theorem for DY probably exists and the Lorentz invariance relations might hold, (3) they are totally different and factorization, as we understand it now, is violated.

It has been advocated[11,13] that fragmentation functions are independent of the link direction. Although the supporting model[13] ignores nonperturbative quarks in the nucleon with $p^2 > m^2$ which could be sensitive to the link direction, the scenario itself remains possible. If fragmentation shows up to be link independent, then factorization in SIDIS is probably feasible. Extended models or experimental evidence (compare z dependences of $D_1(z, P_h^{\perp 2})$ or $H_1^{\perp (1)}(z)$ of SIDIS with e^+e^-) would contribute a lot.

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